

Probability Distributions Cheat Sheet

Type	Name	Notation	Parameters	Probability Density Function (p.d.f.)	Mean	Variance
Discrete	Bernoulli	$X \sim \text{Bern}(p)$	p - probability of success	$P[X=x] = p^x(1-p)^{1-x}$	$E[X] = p$	$\text{Var}[X] = p(1-p)$
	Binomial	$X \sim \text{Bin}(n, p)$	p - probability of success n - number of trials	$P[X=x] = \binom{n}{x} p^x(1-p)^{n-x}$	$E[X] = np$	$\text{Var}[X] = np(1-p)$
	Poisson	$X \sim \text{Poiss}(\lambda)$	λ - mean	$P[X=x] = \frac{\lambda^x e^{-\lambda}}{x!}$	$E[X] = \lambda$	$\text{Var}[X] = \lambda$
Continuous	Normal	$X \sim N(\mu, \sigma^2)$	μ - mean σ^2 - variance	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$E[X] = \mu$	$\text{Var}[X] = \sigma^2$
	Standard Normal	$Z \sim N(0,1)$		$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$	$E[Z] = 0$	$\text{Var}[Z] = 1$
	Exponential	$X \sim \exp(\lambda)$	λ - reciprocal of mean	$f(x) = \lambda e^{-\lambda x}$	$E[X] = \frac{1}{\lambda}$	$\text{Var}[X] = \frac{1}{\lambda^2}$

Results:

- 1) The Binomial Distribution converges to the Poisson Distribution when n is large, p is small and $np < 10$. We have $\lambda = np$.
- 2) The Binomial Distribution converges to the Normal Distribution when n is large and p is close to 0.5. We have $\mu = np$ and $\sigma^2 = np(1-p)$.
- 3) A Normally Distributed random variable $X \sim N(\mu, \sigma^2)$ is converted to a Standard Normally Distributed random variable $Z \sim N(0,1)$ by using the formula:

$$Z = \frac{X - \mu}{\sigma}$$
- 4) Let be X_1, X_2, \dots, X_n be independent, identically distributed random variables with mean μ and variance σ^2 . The **Central Limit Theorem** states that the distribution of the sample mean is: $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \sim N(\mu, \frac{\sigma^2}{n})$ given that n is sufficiently large ($n > 30$). If the population size N is finite, we add the finite population correction factor and the distribution of the sample mean is: $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \sim N(\mu, \frac{\sigma^2(N-n)}{n(N-1)})$.